DN-003-001408 Seat No. ________

B. Sc. (Sem. IV) (CBCS) Examination

April / May – 2015

Mathematics : 401 (A)

Faculty Code : 003
Subject Code : 001408

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

SECTION - I

1 Choose correct answer of the following : **20**

(1) $u = e^{\frac{x}{y}}$ then $\frac{\partial u}{\partial y} =$ ________

(A) 0 (B) $-\frac{x}{y^2} e^{\frac{x}{y}}$

(C) $-\frac{x}{y} e^{\frac{x}{y}}$ (D) $-x\, e^{\frac{x}{y}}$

(2) $u = \tan\left(\frac{x^3 + y^3}{x + y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ________

(A) $2u$ (B) $3u$

(C) u (D) None of these

(3) $f(x, y) = x \cdot \tan\left(\frac{y}{x}\right)$ then $f_x(x, y) =$ ________

(A) $sec^2 \frac{y}{x}$ (B) $\tan\frac{y}{x} - \frac{y}{x^2} sec^2 \frac{y}{x}$

(C) $\tan\frac{y}{x} - \frac{y}{x}\ sec^2 \frac{y}{x}$ (D) $\tan\frac{y}{x} - sec^2 \frac{y}{x}$

(4) $x^3 + y^3 + z^3 - 3xyz = 0$ then $\frac{\partial z}{\partial y} =$ ________

(A) $\frac{zx + y^2}{z^2 - xy}$ (B) $\frac{yz - x^2}{z^2 - xy}$

(C) $\frac{zx + y^2}{z^2 - xy}$ (D) $\frac{yz + x^2}{z^2 - xy}$

(5) If $x = p\cos\theta$, $y = p\sin\theta$ then $\frac{\partial p}{\partial x} =$ ________

(A) $\sec\theta$ (B) $\sin\theta$

(C) $\operatorname{cosec}\theta$ (D) $\cos\theta$

(6) $u = yz$, $v = zx$, $w = xy$ then $\frac{\partial(u,v,w)}{\partial(x,y,z)} =$ ________

(A) 0 (B) xyz

(C) $2xyz$ (D) $2xyz - x^2$

(7) $\frac{\partial(x, y)}{\partial(r, s)} \cdot \frac{\partial(u, v)}{\partial(x, y)} =$ ________

(A) 1 (B) 0

(C) $\frac{\partial(r, s)}{\partial(u, v)}$ (D) $\frac{\partial(u, v)}{\partial(r, s)}$

(8) If $\phi = x^3 + y^3 + z^3$ then grad $\phi =$ ________

(A) $x^2 + y^2 + z^2$ (B) $(2x^2, 2y^2, 2z^2)$

(C) $(3x^2, 3y^2, 3z^2)$ (D) $3x^2 + 3y^2 + 3z^2$

(9) $\overline{f} = (x^3, y^3, z^3)$ then Curl $\overline{f} =$ ________

(A) $3\,\overline{r}$ (B) $3r\overline{r}$

(C) $3r^2\overline{r}$ (D) 0

(10) Vector function $\bar{f}$ is solenoidal then ________

(A) $\nabla \cdot \bar{f} = 0$ (B) $\nabla \times \bar{f} = 0$

(C) $\nabla \bar{f} = 0$ (D) None of these

(11) $\int_0^2 \int_0^2 \left(x^2 + y^2\right) dx\, dy =$ __________

(A) $\frac{32}{3}$ (B) $\frac{16}{3}$

(C) 32 (D) 16

(12) $\int_0^{\sqrt{3}} \int_0^{\sqrt{3}} \frac{dx\, dy}{\left(1+x^2\right)\left(1+y^2\right)} =$ _________

(A) $\frac{\pi}{3}$ (B) $\frac{\pi^2}{3}$

(C) $\frac{\pi^2}{6}$ (D) $\frac{\pi^2}{9}$

(13) In transformation of coordinates from Cartesian to spherical if $x^2 + y^2 + z^2 \le a^2$; $x, y, z \ge 0$ then __________

(A) $0 \le r \le a^2$ (B) $0 \le \theta \le \frac{\pi}{2}$

(C) $0 \le \theta \le \pi$ (D) $0 \le \phi \le \pi$

(14) In transformation, from Cartesian to spherical co-ordinate, $|J| =$_______

(A) r (B) $r \sin^2 \phi$

(C) $r^2 \sin \phi$ (D) None of these

(15) $\lceil p+1 =$ ________

(A) $(p+1)\lceil p$ (B) $p\lceil p+1$

(C) $p!$ (D) None of these

(16) $\beta(3,4) =$ ________

(A) $\frac{1}{60}$ (B) $\frac{1}{30}$

(C) $\frac{1}{15}$ (D) $\frac{1}{12}$

(17) V is inner product space and $u, v \in V$, $\alpha \in R$ then $\|\alpha u\| =$ ________

(A) $|\alpha|\ \|u\|$ (B) $\alpha\ \|u\|$

(C) $|\alpha|\ |u|$ (D) None of these

(18) If a vector u in an inner product space V is orthogonal to it self then u is ________

(A) none zero

(B) zero vector

(C) scalar multiple of some vector

(D) None of these

(19) If u and v are vectors in an inner product space V then Cauchy Schwartz's inequality reduces to equality if u and v are ____________

(A) Zero vectors (B) L.I.

(C) L.D. (D) None of these

(20) R^3 is Euclidean inner product space, $u = (-1, 5, 2)$, $v = (2, 4, -9)$, then angle between u and v is ________.

(A) 1 (B) 0

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

SECTION - II

2 (A) Answer any **three** : **6**

(1) Find f_x and f_y for the following function at (0, 0).

$$f(x, y) = \frac{x^2 - xy}{x + y} \quad : \ (x, y) \neq (0, 0)$$
$$= 0 \quad : \ (x, y) = (0, 0)$$

(2) $w = \frac{y}{z} + \frac{x}{y} + \frac{z}{x}$ then prove that,

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = 0.$$

(3) For implicit function $f(x, y, z) = 0$,

prove that, $\frac{\partial z}{\partial x} = -\frac{f_x}{f_z}$

(4) $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$

then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 10x + 4.$

(5) $\phi(x, y, z) = x^2y + y^2x + z^2$ then find unit normal at (1,1,1).

(6) In usual notation prove that $\nabla\cdot\left(\nabla \times \overline{f}\right) = 0$

2 (B) Answer any **three** : **9**

(1) If $u = \log(\tan x + \tan y + \tan z)$, then

prove that $\sin 2x\frac{du}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$

(2) $u = e^{x^2+y^2+z^2}$ then show that

$$\frac{\partial^3 u}{\partial x\,\partial y\,\partial z} = 8uxyz.$$

(3) If $z = \sin^{-1}\sqrt{\frac{x^3 + y^3}{x^2 + y^2}}$ then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z.$$

(4) If $u = x + y + z,\ uv = y + z,\ uvw = z$ then

find $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$

(5) $\overline{f} = x^2yz\,\hat{i} + xy^2z\,\hat{j} + xyz^2\,\hat{k}$ then find

$(\nabla \times \overline{f})$ at (1, 2, 3).

(6) If $\overline{f}$ is irrotational then find a, b, c

where, $\overline{f} = (2x + 3y + az, bx + 2y + 3z, 2x + cy + 3z)$

2 (C) Answer any **two** : **10**

(1) If u is homogeneous function of x, y of degree n then prove that,

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

(2) If $u = sin^{-1}(x - y);\ x = 3t, y = 4t^3$ then

show that $\dfrac{dy}{dt} = \dfrac{3}{\sqrt{1 - t^2}}$.

(3) In usual notation prove that,

$div(\overline{f} \times \overline{g}) = \overline{g} \bullet Curl\ \overline{f} - \overline{f} \bullet Curl\ \overline{g}$

(4) In usual notation prove that,

$div(r^n\ \overline{r}) = (n + 3)r^n.$

(5) In usual notation prove that,

$$\nabla^2 f(\bar{r}) = f''(r) + \frac{2}{r}f'(r)$$

3 (A) Answer any three : **6**

(1) Find $\iint_R xy\,dx\,dy$, where R is the first quadrant of the circle $x^2 + y^2 = a^2$.

(2) Prove that $\iint_R (x^2 + y^2)\,dx\,dy = \dfrac{\pi}{8}$, where R is bounded by $0 \le y \le \sqrt{1 - x^2}, 0 \le x \le 1$.

(3) Find $\int_{(2, 1)}^{(1, 2)} y\, dx$.

(4) Prove symmetrical property of Beta function.

(5) Prove that $\int_0^2 x^4 \left(8 - x^3\right)^{-1/3} dx = \frac{16}{3} \beta\left(\frac{5}{3}, \frac{2}{3}\right)$

(6) Verify that, $u \cdot v = 2x_1 y_1 + 3x_2 y_2$

where $u = \left(x_1, x_2\right), v = \left(y_1, y_2\right)$ is inner

product on R^2 or not.

3 (B) Answer any three : 9

(1) Prove that $\iint_R e^{x^2 + y^2} dx\, dy = \pi(e - 1)$

where, R is $x^2 + y^2 \le 1$.

(2) Evaluate $\iint_R \left(x^2 + y^2\right) dx\, dy$ where R is triangle

whose vertices are (0, 0), (1, 0) and (1, 1)

(3) Find $\int_C V_n\, ds$ where $V = (3y - x, -2x - y)$ and

C is the line segment joining (0, 0) and (2, 0).

(4) Find $\int_0^{\pi/2} sin^5\theta.\, cos^6\theta\, d\theta$.

(5) Prove that $\int_0^\infty \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}} dx = 2\beta(m, n)$

(6) State and prove Triangle Inequality.

3 (C) Answer any two : **10**

(1) Find $\iint_R \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\, dx\, dy$, where R is the first quadrant of unit circle.

(2) State and prove green's theorem.

(3) Verify the green's theorem for line integral $\oint_C x^2 y\, dx + xy^2\, dy$, where C is the first quadrant bounded by $y = x$ and $y^3 = x^2$.

(4) In usual notation prove that $\beta(p, q) = \frac{\Gamma p\, \Gamma q}{\Gamma (p+q)}$

(5) Let R^3 have the Euclidean inner product.
Use Gram-Schmidt process to transform the basis $\{(1, 1, 1), (-1, 1, 0)(1, 2, 1)\}$ into orthonormal basis.